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FILM CONDENSATION OF MULTICOMPONENT MIXTURES

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Abstract—The classic Nusselt model is generalized for the condensation process of a multicomponent vapor mixture. The condensed vapors may be miscible in their liquid state or contain noncondensable gases.

The reduction in the condensation rate owing to the accumulation of a noncondensable gas or the more volatile components near the condensate interface is demonstrated for three component systems of methanol–water–air and acetone–methanol–water. Also the effects of interfacial suction and forced convection are included.

The analytical solution incorporates Diffusion Law for a multicomponent system and both exact and approximate integral method solutions are applied. The accuracy of the integral method turns out to be remarkably good.

1. INTRODUCTION

This work is concerned with the condensation process of a multicomponent vapor mixture on a cooled vertical plate of constant temperature. The vapors which condense in a form of a laminar gravity flow are considered completely miscible in their liquid state. However, the inclusion of noncondensable gases is also possible.

A considerable amount of work has been devoted to develop analytical means for predicting condensation rate of a single component in the presence of noncondensable gas, Sparrow & Lin (1964); Minkowycz & Sparrow (1967); Rose (1969); Taitel & Tamir (1969, 1971). Condensation of a binary condensable mixture has also been treated by Colburn & Drew (1937); Sparrow & Marshall (1969) and recently by Denny & South (1972) and Denny & Jusionis (1972). However, it seems that previous efforts were mainly restricted to the treatment of binary mixtures. In this article, we extend the analysis and generalize it to a unified treatment of any number of components whether condensable or contain noncondensable gases.

2. ANALYSIS

2.1 *Formulation*

The physical model and coordinate system are shown in figure 1. A stagnant multicomponent mixture at temperature T_∞ and mass concentration $w_{i\infty}$, $i = 1 \dots n$, is con-

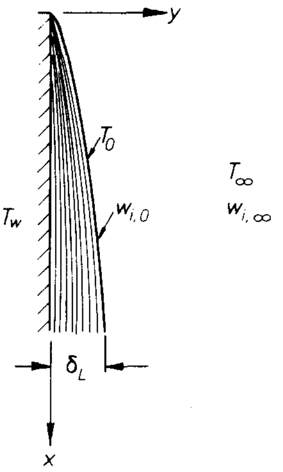

Figure 1. Physical model and coordinate system.

densed on a vertical wall which is maintained at a constant temperature T_w. Condensation occurs in the form of a thin laminar film of the condensate mixture flowing by gravity downwards. External to the liquid film, boundary layers of velocity, temperature and concentrations are formed. The velocity profile in the gas phase is caused by the interfacial shear at the moving liquid film and by gravity induced flow due to density variations in the vapor boundary layer.

The transport phenomena in the vapor boundary layer are controlled by the conservation equations of mass, momentum, energy and the conservation of each chemical species. In this analysis we assume that the properties are constant. Nevertheless, property variation is partially accounted for by calculating them in an appropriate average reference state. Hence, the following equations control the conservation of mass and momentum:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad [1]$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\left(1 - \frac{\rho_\infty}{\rho}\right) + \nu\frac{\partial^2 u}{\partial y^2}, \qquad [2]$$

where x, y are the cartesian coordinates and u, v are their respective velocity components. g is the acceleration of gravity, ρ is the density and ν is the kinematic viscosity. The subscript ∞ refers to bulk conditions. Since the sensible heat is relatively unimportant in condensation problems (Sparrow & Marschall 1969) the energy equation is discarded. The term $g(1 - \rho_\infty/\rho)$ in [2] is the buoyancy force. As customary in free convection problems, the density in this term is not constant although in general, properties are considered constant. The ratio ρ_∞/ρ for small temperature differences ($T/T_\infty \simeq 1$) and ideal gas behaviour is given by:

$$\frac{\rho_\infty}{\rho} = \frac{M_\infty}{M} = \frac{\sum_{j=1}^{n} w_j/M_j}{\sum_{j=1}^{n} w_{j\infty}/M_j}, \qquad [3]$$

where M is the molecular weight and w_j is the mass fraction of component j.

The conservation of each chemical species in the boundary layer is described by the diffusion equation. We use here multicomponent diffusion law given, for example, by Bird *et al.* (1960, p. 569). However it is convenient to cast this equation of transfer in terms of the mass concentration. In this case we obtain

$$\mathbf{m}_i = \rho \sum_{j=1}^{n} D_{ij}\nabla w_j + \rho \mathbf{v} w_i, \qquad [4]$$

where $\mathbf{m}_i$ and w_i are the mass flux and the mass fraction of component i. $\mathbf{v}$ is the velocity vector and the coefficient D_{ij} is given by:

$$D_{ij} = M_i D_{ij}^M \sum_{K=1}^{n} \frac{w_K}{M_K} - \frac{M_i}{M_j} \sum_{K=1}^{n} D_{iK}^M w_K, \qquad [5]$$

D_{ij}^M is the multicomponent diffusion coefficient that can be evaluated using Hirschfelder *et al.* (1964, p. 541). For a ternary mixture a simple equation is available in Bird *et al.* (1960, p. 570).

Using [4], the boundary layer diffusion equation takes the form

$$u\frac{\partial w_i}{\partial x} + v\frac{\partial w_i}{\partial y} = -\sum_{j=1}^{n} D_{ij}\frac{\partial^2 w_j}{\partial y^2} \qquad i = 1 \ldots n. \qquad [6]$$

Again, we assume constant properties, namely, ρ and D_{ij} are considered constant provided their value is evaluated at a suitable reference state. In spite of the fact that the coefficient D_{ij} is concentration dependent, it has been shown by Taitel & Tamir (1974) that the use of constant coefficient evaluated at the reference state is quite accurate.

Equations [1], [2] and [6] admit a similarity transformation as follows:

The independent variable η is defined as

$$\eta = cy/x^{1/4}; \qquad c = (g/4\nu^2)^{1/4}. \qquad [7]$$

The dependent variables are:

$$f(\eta) = \frac{\psi}{4\nu c x^{3/4}} \qquad [8]$$

where ψ is the usual stream function such that

$$u = 4\nu c^2 x^{1/2} f', \qquad [9]$$

$$v = \nu c x^{-1/4}(cyx^{-1/4}f' - 3f) \qquad [10]$$

and
$$\phi_i(\eta) = \frac{w_i - w_{i\infty}}{w_{io} - w_{i\infty}}. \qquad [11]$$

Using these transformations and eliminating component p by using $\sum_{k=1}^{n} w_k = 1$, [1], [2] and [6] are reduced into:

$$f''' + 3ff'' - 2f'^2 + \sum_{\substack{j=1 \\ j \neq p}}^{n} \Gamma_j \phi_j = 0, \qquad [12]$$

$$3f\phi_i' + \sum_{\substack{j=1 \\ j \neq p}}^{n} \frac{D_{ip} - D_{ij}}{\nu} \psi_{ij} \phi_j'' = 0 \qquad i \neq p \qquad [13]$$

where
$$\Gamma_j = \left(\frac{1}{M_p} - \frac{1}{M_j}\right) \frac{w_{jo} - w_{j\infty}}{\sum_{K=1}^{n} w_{K\infty}/M_K}; \; \psi_{ij} = \frac{w_{jo} - w_{j\infty}}{w_{io} - w_{i\infty}}. \qquad [14]$$

The boundary conditions for the solution of [12] and [13] are rather complex at the liquid–vapor interface. The only simple determination is $\phi_i(0) = 1$. However, note that w_{io} is yet unknown. Detailed look upon the interface matching conditions will be given later. The boundary conditions at the bulk of the vapor are given as follows:

$$f'(\infty) = 0; \qquad \phi_i(\infty) = 0. \qquad [15]$$

The zero value of $f'(\infty)$ states that the velocity in the x-direction is zero (see [9]). It is possible however to get a solution for a non-zero velocity at infinity, but in order to have similarity solution it must be proportional to $x^{1/2}$.

The liquid condensate is treated no differently than the old classic Nusselt model. The continuity, momentum and energy equations, in their commonly used simplified form read:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad \rho_L g + \mu_L \frac{\partial^2 u}{\partial y^2} = 0, \qquad \frac{\partial^2 T}{\partial y^2} = 0, \qquad [16]$$

and the boundary conditions are:

$$\begin{aligned} u = 0; \quad v = 0; \quad T = T_w \quad & \text{at } y = 0 \\ \partial u/\partial y = 0; \quad T = T_o \quad & \text{at } y = \delta_L(x) \end{aligned} \qquad [17]$$

where again, fluid properties are to be taken at an appropriate reference state. The temperature at the interface $y = \delta_L$, is constant and equal to T_o. This is consistent with the solution in the vapor region which yields a similarity transformation for a constant interfacial temperature. The solution of [16] is well documented and we present only the relevant results.

$$-\frac{\dot{m}x}{\mu_L} = \left[\frac{Cp_L(T_o - T_w)}{\lambda Pr_L}\right]^{3/4}\left(\frac{gx^3}{4\nu_L^2}\right)^{1/4}, \qquad [18]$$

$$\frac{u_o x}{\nu_L} = 2\left[\frac{Cp_L(T_o - T_w)}{\lambda Pr_L}\right]^{1/2}\left(\frac{gx^3}{4\nu_L^2}\right)^{1/2}, \qquad [19]$$

$$\frac{\delta_L}{x} = \left[\frac{Cp_L(T_o - T_w)}{\lambda Pr_L}\right]^{1/4}\left(\frac{gx^3}{4\nu_L^2}\right)^{-1/4}. \qquad [20]$$

Here Cp is the specific heat at constant pressure, λ is the latent heat and Pr is the prandtl number. The subscript L refers to the condensed liquid, o to the interface and w to the plate surface.

The boundary conditions of the vapor boundary layer are now matched with the solution of the condensate film at $y = \delta_L$. For simplicity, and in order to be consistent with our approximate integral method solution, we identify $y = \delta_L$ for the liquid region with $y = o$ for the vapor region. The solution and results are however, completely identical had we taken the liquid interface at $y = \delta_L$ instead of $y = o$.*

We consider now the matching conditions at the interface.

(a) *Total mass flux.* The total mass flux towards the interface (flux towards the interface is negative), using [10], is:

$$-\dot{m} = -\rho v = 3\mu c x^{-1/4} f_o. \qquad [21]$$

When this is matched with [18] one gets:

$$f_o = \frac{1}{3}\left(\frac{\rho_L \mu_L}{\rho\mu}\right)^{1/2}\left[\frac{Cp_L(T_o - T_w)}{\lambda Pr_L}\right]^{3/4}. \qquad [22]$$

(b) *Tangential velocity.* Equating [9] and [19] yields continuity of the tangential velocity when

$$f_o' = \frac{1}{2}\left[\frac{Cp_L(T_o - T_w)}{\lambda Pr_L}\right]^{1/2}. \qquad [23]$$

(c) *Conservation of the flux of the individual chemical components.* The flux of any species is given by [4]. The ratio between $\dot{m}_i$ and the total flux $\dot{m} = \rho v$ must be equal to the liquid mass fraction of this component. Thus, using [4] and [10], we obtain

$$\bar{w}_i = w_{io} + \frac{1}{3f_o}(w_{io} - w_{i\infty}) \sum_{\substack{j=1 \\ j \neq p}}^{n} \frac{D_{ip} - D_{ij}}{\nu} \psi_{ij}\phi_j' \qquad i \neq p. \qquad [24]$$

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* In this case our similarity variable, η, would take the form $\eta = c(y - \delta_L)/x^{1/4}$. With the aid of [20] we would obtain exactly [12] and [13]. Also one may observe that in our simplified approach we calculated convective flow towards interface by the term $-\dot{m} = -\rho v$ whereas when the thickness δ_L is taken into account one should use $-\dot{m} = \rho(u\, d\delta_L/dx - v)$. However both terms are the same in terms of the similarity variables and equals $-\dot{m} = 3\mu c x^{-1/4} f(o)$.

Note that the concentration of each component in the liquid film is uniform. Thus there is no question of diffusion within the liquid film.

(d) *Equilibrium conditions.* We assume that equilibrium conditions exist at the interface. Thus if we designate by X_i the mole fraction of component i in the liquid and by Y_i the mole fraction of component i in the vapor, the following equation holds:

$$X_i = \frac{Y_i P}{\gamma_i P_i^o(T)} \qquad [25]$$

where $P_i^o(T)$ is the vapor pressure of the pure chemical component and γ_i is the activity coefficient of component i in the liquid. For ideal mixtures $\gamma_i = 1$ but usually, γ_i is a function of the mole fraction of all components. Sometimes pressure and temperature dependence can also be accounted for. In our approach we did not make any distinction between condensable vapor and noncondensable gas (or gases). Indeed if one wishes to consider noncondensable gas, all that is needed is to equate its activity coefficient to infinity. Equation [25] when satisfied, will result in a zero mole fraction X_i (or mass fraction $\bar{w}_i$) in the liquid film. The left hand side of [24] will be zero, and thus the usual condition for the impermeability of the noncondensable gas is automatically achieved.

As previously mentioned we need to consider properties in their reference condition. In this we followed Sparrow's (1969) suggestion as follows: The liquid reference temperature was taken at

$$T_L^* = T_w + \tfrac{1}{3}(T_o - T_w). \qquad [26]$$

All liquid properties were calculated at this temperature except for the latent heat of evaporation which was calculated at the actual interface temperature (concentration is constant in the liquid film).

The vapor reference temperature and concentration were taken as

$$T^* = \tfrac{1}{2}(T_o + T_\infty), \qquad w_i^* = \tfrac{1}{2}(w_{io} + w_{i\infty}). \qquad [27]$$

2.2 *Method of solution*

A general computer program was written which can handle any number of desired chemical components. The solution is carried along the following steps:

(a) A first guess of the interfacial temperature T_o and the concentration w_{io} of the $(n-2)$ out of the n chemical components is inserted (total pressure is given).

(b) Using equilibrium conditions, one can calculate all values of mass fraction at the interface, w_{io}, and liquid, $\bar{w}_{io}$. Equation [25] provides n equations for the $n + 2$ unknowns X_i's and two unknown Y_i's. Additionally two equations are provided by $\sum X_i = \sum Y_i = 1$. Thus by solving $(n + 2)$ linear equations all concentrations are calculated. This calculation, however, is accompanied with successive iterations. Firstly because the activity coefficients γ_i's are mole fraction dependent, and secondly because we assume the $(n - 2)$ w_i's to be known, rather than the Y_i's.

(c) Once T_o, w_{io}, $\bar{w}_i$ are known (in addition to P, T_w, T_∞ and $w_{i\infty}$ which are apriori fixed) all properties are calculated in their suitable reference temperature and concentration. Also the values Γ_j [12, 14], ψ_{ij} [13, 14], f_o [22] and f'_o [23] are calculated.

(d) Next the n simultaneous ordinary differential equations [12] and [13] are solved numerically by the method of Runge–Kutta. To this effect, initial conditions for f_o, f'_o and $\phi_{io} = 1$ are known. Unfortunately rather than the initial values of f''_o and ϕ'_{io}, the values of $f' = 0$ and $\phi_i = 0$ at $\eta = \infty$ (practically large η) are given. Thus, the values of f''_o and ϕ'_{io} are initially guessed. Very fast convergence is achieved by the use of Newton–Raphson iteration technique (Perry 1963) when considering the functions

$$
\begin{aligned}
f'_\infty &= F_1(f''_o, \phi'_{io}) \\
\phi_{i\infty} &= F_{2i}(f''_o, \phi'_{io})
\end{aligned}
\qquad [28]
$$

which should approach zero. The partial derivatives needed in this technique were calculated numerically.

(e) The values of the $(n-1)\bar{w}_i$ ($\bar{w}_p$ is excluded) are now calculated by [24]. These values should be equal to those obtained from equilibrium conditions in step (a). Again we use Newton–Raphson technique in order to force fast convergence. $(n-1)$ functions are considered

$$
z_i = z_i(T_o, w_{io}) = \bar{w}_i(\text{step a}) - \bar{w}_i(\text{step e}) \qquad i \neq p. \qquad [29]
$$

These functions depend on our $(n-1)$ initial guesses, T_o and w_{io}. Once z_i is zero, T_o and w_{io} are the sought final result.

2.3 *Heat transfer*

The main aim of our calculations is to estimate the heat transfer to the wall or equivalently, the condensation rate. Results are presented in a form of heat transfer efficiency, q/q_o, which is the ratio between the local heat flux to the wall q, and q_o. q_o is a reference heat flux calculated for $w_{io} = w_{i\infty}$ and $T_o = T_\infty$, namely when condensation occurs at bulk conditions.

This efficiency is given by

$$
\frac{q}{q_o} = \frac{\lambda(\rho_L\mu_L)^{1/2}\left[\dfrac{Cp_L(T_o - T_w)}{\lambda Pr_L}\right]^{3/4}}{\left\{\lambda(\rho_L\mu_L)^{1/2}\left[\dfrac{Cp_L(T_o - T_w)}{\lambda Pr_L}\right]^{3/4}\right\}_{T_o = T_\infty,\, w_{io} = w_{i\infty}}} \qquad [30]
$$

where the properties of the denominator are calculated for $T_o = T_\infty$ and $w_{io} = w_{i\infty}$.

2.4 *The integral method*

In addition to the exact solution which is relatively complicated and time consuming, the approximate integral method is also applied. In this method we assume polynomial profiles for the velocity and concentrations and thereby reduce the set of partial differential equations into a set of ordinary differential equations. These equations are solved analytically and transform the differential equations into a set of algebraic equations. Although the use of a digital computer is also essential here, the solution is simpler and much less time consuming than the exact solution.

The momentum and diffusion integral equations are obtained by integrating [1], [2] and [6] which yields:

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_o^{\delta}\left(u^2 - u_\infty u\right)\mathrm{d}y - v_o(u_o - u_\infty) = -\nu\left.\frac{\partial u}{\partial y}\right)_o - g\theta\sum_{\substack{j=1\\ j\neq p}}^{n}\left(\frac{1}{M_j} - \frac{1}{M_p}\right)\int_o^{\delta}(w_j - w_{j\infty})\,\mathrm{d}y \tag{31}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_o^{\delta_i}\left(u(w_i - w_{i\infty})\,\mathrm{d}y - v_o(w_{io} - w_{i\infty}) = -\sum_{\substack{j=1\\ j\neq p}}^{n}(D_{ip} - D_{ij})\left.\frac{\partial w_j}{\partial y}\right)_o\right. \tag{32}$$

where δ is the velocity boundary layer thickness, δ_i is the concentration boundary layer thickness and

$$\theta = \frac{1}{\sum\limits_{\substack{j=1\\ j\neq p}}^{n}\left(\frac{1}{M_j} - \frac{1}{M_p}\right)w_{j\infty} + \frac{1}{M_p}}\,.$$

In [31] the term for the buoyancy force [3] is already explicitly inserted and also component p is eliminated from the calculations.

The velocity profile is assumed to have the form

$$u = u_\infty + (u_o - u_\infty)\left(1 - \frac{y}{\delta}\right)^2 + (\tilde{u} - u_\infty)\frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2 \tag{34}$$

and for the concentration profile we assume

$$w_i = w_{i\infty} + (w_{io} - w_{i\infty})\left(1 - \frac{y}{\delta_i}\right)^2. \tag{35}$$

Note that [31] and [34] do contain the possibility that the bulk of vapor is not stagnant but may have a velocity $u_\infty = u_\infty(x)$.

Upon inserting [34] and [35] into [31] and [32] we obtain the following momentum and diffusion equations:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\{\delta[\tfrac{1}{3}u_\infty(u_o - u_\infty) + \tfrac{1}{12}u_\infty(\tilde{u} - u_\infty) + \tfrac{1}{15}(u_o - u_\infty)(\tilde{u} - u_\infty) + \tfrac{1}{105}(\tilde{u} - u_\infty)^2 \\ + \tfrac{1}{5}(u_o - u_\infty)^2]\} - v_o(u_o - u_\infty) + \frac{\nu}{\delta}[\tilde{u} - 2u_o + u_\infty] \\ + g\theta\delta\left\{\sum_{\substack{j=1\\ j\neq p}}^{n}\left(1 + \frac{1}{3\xi_j^2} - \frac{1}{\xi_j}\right)\left(\frac{1}{M_j} - \frac{1}{M_p}\right)(w_{jo} - w_{j\infty})\right\} = 0\end{aligned} \tag{36}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\tfrac{1}{3}\delta(w_{io}-w_{i\infty})\left[u_\infty\xi_i+(u_o-u_\infty)(\xi_i-\tfrac{1}{2}\xi_i^2+\tfrac{1}{10}\xi_i^3)+(\tilde{u}-u_\infty)(\tfrac{1}{4}\xi_i^2-\tfrac{1}{5}\xi_i^3+\tfrac{1}{20}\xi_i^4)\right]\right\}$$

$$-v_o(w_{io}-w_{i\infty})=\frac{2}{\delta}\left\{\sum_{\substack{j=1\\j\neq p}}^{n}\frac{1}{\xi_j}(D_{ip}-D_{ij})(w_{jo}-w_{j\infty})\right\}. \qquad [37]$$

The solution of [36] and [37] is subjected to the matching of the total mass flux [18], tangential velocity [19], equilibrium at the liquid–vapor interface [25] and the flux of every chemical component which, by taking into account that $\dot{m}_i/\dot{m}=\bar{w}_i$, yields

$$(w_{io}-\bar{w}_{io})(v_o+v_s)+\sum_{\substack{j=1\\j\neq p}}^{n}(D_{ij}-D_{ip})\left.\frac{\partial w_j}{\partial y}\right)_o=0 \qquad \begin{array}{l} i=1\ldots n. \\ i\neq p \end{array} \qquad [38]$$

Note that [38] contains the possibility of interfacial suction which is represented by the suction velocity $v_s(x)$.

This set of equations has a solution when:

$$\delta=A\left(\frac{v^2}{4g}\right)^{1/4}x^{1/4} \qquad [39]$$

$$\tilde{u}=Bg^{1/2}x^{1/2} \qquad [40]$$

$$u_\infty=Qg^{1/2}x^{1/2} \qquad [41]$$

$$v_s=Kx^{-1/4} \qquad [42]$$

and if T_o, w_{io} and ξ_i are constant. Q and K are constant parameters which characterize the bulk velocity and suction velocity respectively. Inserting [39] and [42] into the momentum equation [36], and using the values for v_o and u_o from [18] and [19] yields

$$35Q(S^{1/2}-Q)+\tfrac{35}{4}Q(B-Q)+7(S^{1/2}-Q)(B-Q)+(B-Q)^2$$

$$+21(S^{1/2}-Q)^2+\frac{84}{A}S^{3/4}G^{1/2}(S^{1/2}-Q)+\frac{168}{A^2}(B-2S^{1/2}+Q)$$

$$+84\theta\left\{\sum_{\substack{j=1\\j\neq p}}^{n}\left(1+\frac{1}{3\xi_j^2}-\frac{1}{\xi_j}\right)\left(\frac{1}{M_j}-\frac{1}{M_p}\right)(w_{jo}-w_{j\infty})\right\}=0=F \qquad [43]$$

where $$S=\frac{Cp_L(T_o-T_w)}{\lambda Pr_L} \qquad G=\frac{\mu_L\rho_L}{\mu\rho}.$$

Equation [37] takes the form

$$\xi_iA^2Q+A^2(S^{1/2}-Q)(\xi_i-\tfrac{1}{2}\xi_i^2+\tfrac{1}{10}\xi_i^3)+A^2(B-Q)(\tfrac{1}{4}\xi_i^2-\tfrac{1}{5}\xi_i^3+\tfrac{1}{20}\xi_i^4)$$

$$+4AS^{3/4}G^{1/2}-16\left\{\sum_{\substack{j=1\\j\neq p}}^{n}\frac{1}{\xi_j}\,\frac{D_{ip}-D_{ij}}{v}\,\frac{w_{jo}-w_{j\infty}}{w_{io}-w_{i\infty}}\right\}=0=G_i \qquad [44]$$

and [38]

$$(w_{io} - \bar{w}_{io})(f_s - 1) - \frac{4}{A} S^{-3/4} G^{-1/2} \left[\sum_{\substack{j=1 \\ j \neq p}}^{n} \frac{1}{\xi_j} \frac{D_{ij} - D_{ip}}{\nu} (w_{jo} - w_{j\infty}) \right] = 0 \qquad [45]$$

where f_s is the ratio between the suction rate and the total vapor flux reaching the interface and is given by

$$f_s = K S^{-3/4} G^{-1/2} (g\nu^2/4)^{-1/4}. \qquad [46]$$

Equations [43]–[45] and the equilibrium conditions [25] consist of a set of algebraic equations for the unknown parameters A, B, S (or T_o), ξ_i, w_{io}, $\bar{w}_i$ $(i \neq p)$. Since the number of unknowns is larger than the number of equations we must further assume that one of the concentration boundary layer thickness equals the velocity boundary layer thickness, namely $\delta_m = \delta$ or $\xi_m = 1$.

One expects that the component which tends to diffuse most easily has the largest boundary layer thickness. This choice is made by considering the numerical values of the binary diffusivities D^B_{ij}. We choose the component which appears with the highest diffusion coefficients and designate it as component m.

With the aid of the equilibrium routine (described in "step b" of the exact solution) and after eliminating some of the unknowns by direct substitutions, one ends up with $(n - 1)$ non linear algebraic equation and $(n - 1)$ unknowns. The unknown variables are chosen as T_o and $(n - 2)$ w_{oj}'s. The algebraic equations are [43] and [44] which represent $(n - 2)$ equations ([44] is not used here for $i = p$ and $i = m$). The variable A and ξ_i are eliminated from [43] and [44] by the solution of the set of linear equations [45] for the $1/\xi_i$ and A in terms of the variables which depends only on the unknown variables T_o and w_{io}'s. Equation [44] for the case of $i = m$, namely $\xi_i = 1$, is used to eliminate the variable B from [43]. Thus, the solution procedure starts similar to the exact solution, with a first guess of T_o and $(n - 2)w_{io}$'s as in step (a) of the exact solution. We follow then step (b) and (c) like in the exact solution. After that, the function F (equation [43]) and G_i (equation [44] for $i \neq p$, $i \neq m$) are calculated. The Newton–Raphson iteration technique is used to force F and G_i to approach zero. Although this solution may seem somewhat cumbersome it is handled very easily by a computer and results are obtained using much less computer time as compared with the exact solution.

3. RESULTS AND DISCUSSION

Results are reported for two ternary systems. The first system consists of two condensable vapors, methanol and water in the presence of a noncondensable gas–air. The second system consists of three condensable vapors, acetone, methanol and water.

All results are at total pressure of 760 mmHg. Bulk conditions were assumed to be at the saturation state corresponding to the ambient temperature T_∞ and a prescribed mass fraction of one component. The concentrations of the two other components are determined from equilibrium conditions as consistent with the phase rule.

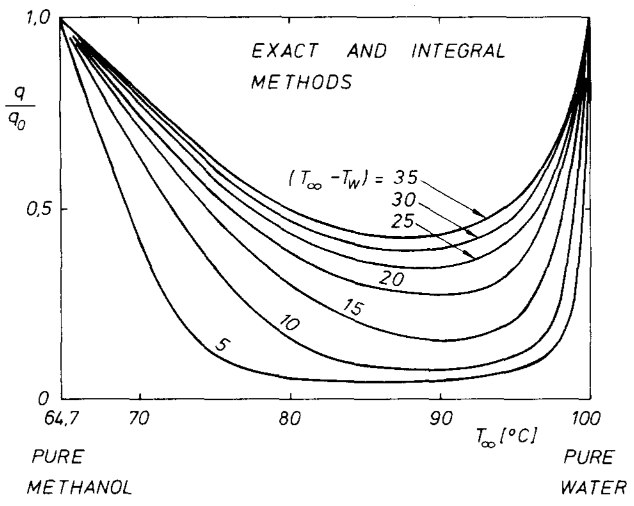

Figure 2. Heat transfer efficiency, methanol–water.

The references Hala *et al.* (1968, 1958); McAdams (1954); Perry (1963); Reid & Sherwood (1966) and VDI Warmeatals (1953) were consulted for the selection of properties. The properties of the individual components and of the mixtures were correlated by algebraic equations whereby temperature and concentration dependence were adequately represented in the range of interest. These properties were then determined in their proper reference temperature and concentration as outlined in the preceding sections.

The first system can be interpreted as a study of the effect of a noncondensable gas on the condensation of a binary condensable mixture. Figure 2 represents, firstly, the results of q/q_o for the binary mixture of methanol–water. The results for this binary mixture were reported in the past by Sparrow & Marshall (1969), although for somewhat different operating conditions and in a different form. We did repeat these calculations and received results which do not differ from Sparrow's results by more than 5 per cent. This discrepancy is attributed to the uncertainty in the selection of the physical properties and the method used in their correlation.

In figure 2 the ratio q/q_o is plotted as a function of the bulk temperature, T_∞, where the temperature difference driving force, $(T_\infty - T_w)$, is the parameter. As can be seen the ratio q/q_o is unity for pure methanol ($T_\infty = 64.7°C$) and pure water ($T_\infty = 100°C$). In the region between the boiling temperatures of both liquids the condensation efficiency, as measured by the ratio q/q_o, drops considerably. This phenomenon can be explained as follows: When the mixture is carried towards the cold wall and condenses, the rate of condensation of the less volatile liquids is higher than that of the volatile ones. Therefore the volatile components accumulate near the liquid–vapor interface in a manner similar to the accumulation of a noncondensable gas. Thus, at the interface the concentration of the volatile liquid increases and therefore the equilibrium temperature at the interface, T_o, decreases, thereby causing a reduction in heat transfer and condensation rate. One may also observe that this reduction is appreciable when the temperature difference $(T_\infty - T_w)$

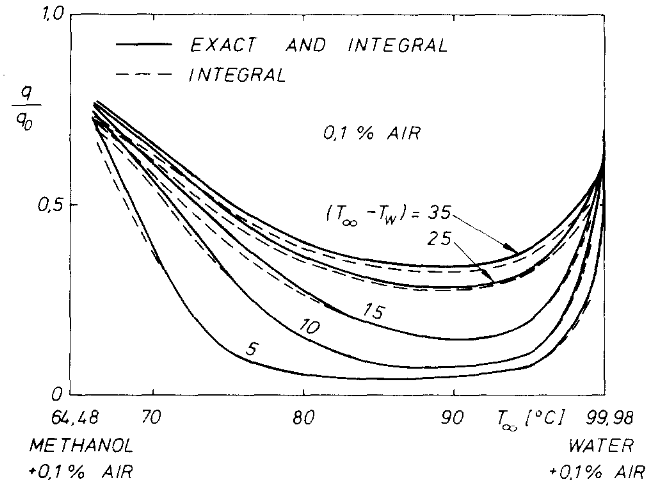

Figure 3. Heat transfer efficiency, methanol–water–air (0.1 %).

is small. This is somewhat contrary to the case of one component in the presence of a noncondensable gas in which case (at least for water–air mixture) the effect of the noncondensable gas is generally higher at larger temperature differences.

The effect of the noncondensable air with bulk concentration of 0.001, 0.01 and 0.1 is given in figures 3–5 respectively. The bulk temperature in these figures ranges from equilibrium conditions of pure methanol with the prescribed amount of air, to the equilibrium condition of water with the same amount of air. As expected, there is a considerable reduction in the condensation rate for increasing concentration of the noncondensable gas in the bulk atmosphere. This effect is larger at higher temperature differences and for the case where one of the components, water or methanol, is in low concentration. In other

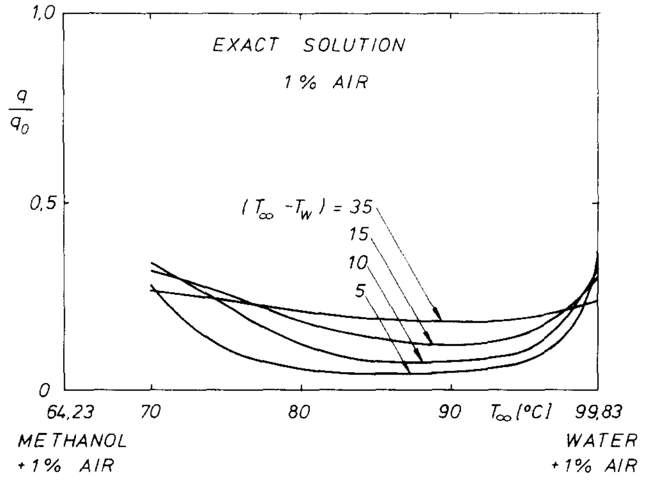

Figure 4. Heat transfer efficiency, methanol–water–air (1 %).

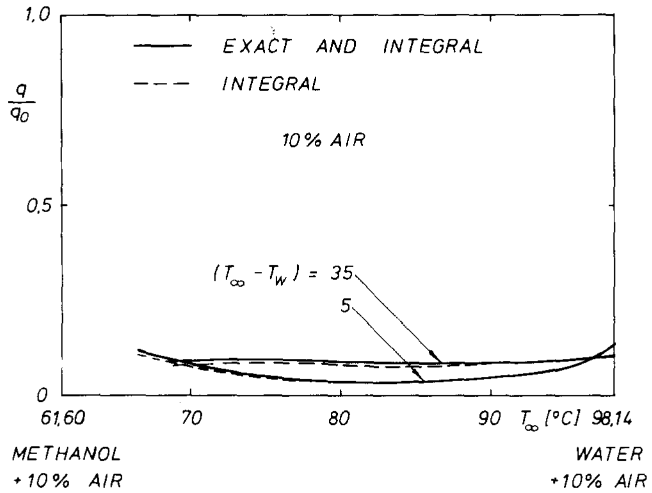

Figure 5. Heat transfer efficiency, methanol–water–air (10%).

words, the presence of the noncondensable gas, affects a single component (or a mixture with a high concentration of a single component) more than a mixture where the concentration of neither volatile component is nearly pure. One may also observe that unlike the case of a binary mixture (figure 2) where q/q_0 always increases for increasing, $(T_\infty - T_w)$, the existence of air causes the reverse effect in a nearly pure water/air and methanol/air mixture.

One may notice that the curves of figures 3–5 are not complete, namely, they do not contain the information for q/q_0 at the left side of the abscissa where the mixture is rich with methanol. Apparently a mathematical solution of our equations in this range does

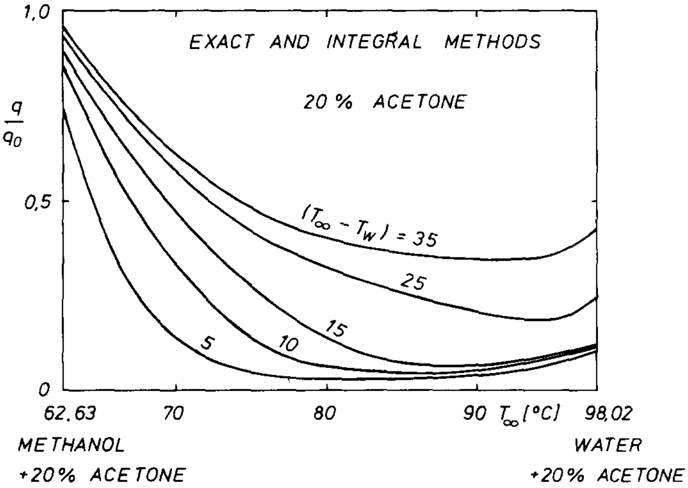

Figure 6. Heat transfer efficiency, acetone (20%)–methanol–water.

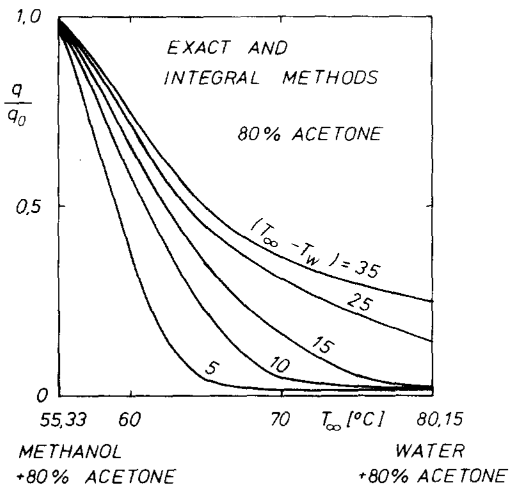

Figure 7. Heat transfer efficiency, acetone (80 %)–methanol–water.

not exist. Furthermore, as commented by Rose (1969), the boundary layer equations are invalid for the case where the molecular weight of the noncondensing gas is smaller than that of the vapor. This is because the accumulation of the light noncondensable gas causes an upward velocity at some distance from the falling liquid interface due to the buoyancy forces. Thereby, the condition $u = 0$ and $x = 0$ can not be fulfilled and our formulation becomes invalid.

The results for the second example of three condensable mixtures are summarized in figures 6 and 7 for bulk conditions of acetone of 20 and 80 per cent mass fraction, respectively. The operating conditions range, as before, from 5 to 35 temperature difference driving force, whereas the bulk temperature covers the whole range from pure methanol with acetone to pure water with acetone. Note that for this system a mathematical solution always exists because the more volatile liquids have higher molecular weights and thus conditions of reverse, upward flow cannot occur.

By comparing the results in figures 6 and 7 with those of figure 2, one can observe that introducing a third volatile component, namely, acetone into the methanol–water mixture causes a further decrease in the heat transfer efficiency (for a constant ambient temperature and driving force) as compared with the binary case. The extent of this reduction is, however, much less than the one caused by a noncondensable gas. Thus, for example, 1 per cent air is more "effective" in reducing the heat transfer efficiency than 20 per cent acetone. Since acetone and methanol do not differ much in their boiling point temperature, the reduction in the heat transfer efficiency for acetone–methanol mixture is much less than for acetone–water mixture. Furthermore, for the case of 80 per cent acetone and 20 per cent methanol at bulk condition, a binary mixture is formed at the interface which is nearly an azeotropic system. This causes the heat transfer efficiency to be very close to unity.

It is interesting to compare the results obtained by the exact solution and those obtained by the simplified integral method solution. For the case of a binary mixture of methanol–water (figure 2) and a ternary mixture of acetone–methanol–water (figures 6 and 7), the

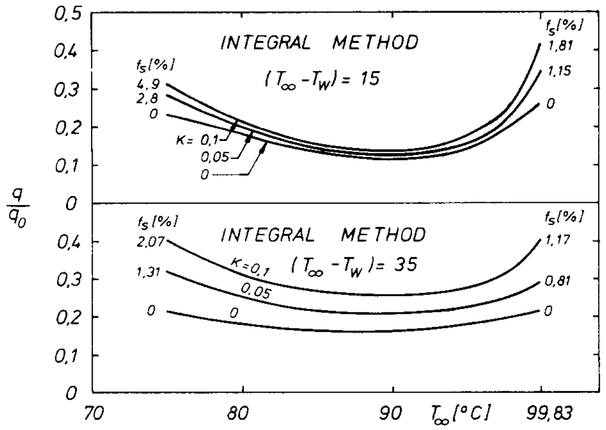

Figure 8. Improving condensation rate by interfacial suction, methanol–water–air (1 %).

accuracy of the integral method is indeed remarkable. The discrepancy between the two methods is so small that it cannot be detected on these figures. Comparison between the two methods for the system of methanol–water–air (figures 3 and 5) shows that also for this system the accuracy of the integral method is satisfactory, in particular at low temperature driving forces. This suggests that the formulation of the integral method can be used quite safely instead of the exact solution.

By applying interfacial suction and/or forced convection it is possible to improve the condensation rate. We examined this effect for the case of methanol–water mixture with 1 per cent air. Note, however, that this is done only for the special case where the suction velocity, v_s, is proportional to $x^{-1/4}$ and u_∞ is proportional to $x^{1/2}$. Otherwise a similarity solution is not possible. Although such special cases may not be the most realistic way of applying forced convection and suction it may give us a feel as to the extent of the effect involved.

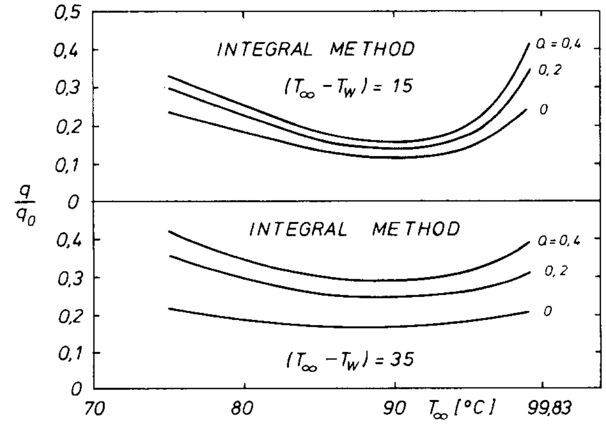

Figure 9. Improving condensation rate by forced convection, methanol–water–air (1 %).

Figure 8 demonstrates the effect of suction for the case of 15 and 35°C temperature difference and for different values of the suction parameter K. For constant K the ratio between the suction rate and total vapor flux, f_s, is not constant. Therefore its value is given at the two end points for each curve. As can be seen, even a small amount of suction rate as much as 0.5–2 per cent, may cause a considerable amount of increase in the condensation rate. This effect is accentuated for larger temperature difference driving force. Also the effect of suction increases as we move towards a binary mixture of water/air or methanol/air.

The effect of forced convection is demonstrated in figure 9 for the same conditions as in the preceding paragraph. The value of the convective velocity is given by the dimensionless parameter Q. As can be seen, also forced convection may cause considerable improvement in condensation rate.

4. SUMMARY AND CONCLUSION

In this work we present a model and the mathematical theory which enables us to predict the condensation rate for a multicomponent vapor mixture on a vertical plate of constant temperature. This theory is based solely on the conservation equations and is free of empirical constants. The results are obtained by two different methods: an exact similarity solution of the conservation equations and by the approximate integral method. The agreement between the two methods is very good.

When a multicomponent mixture condenses on a cold vertical plate, the temperature at the vapor–liquid interface becomes less than the temperature in the bulk. This is caused by the accumulation of the more volatile components or noncondensable gases at the interface, thereby reducing the condensation rate. The reduction in the condensation rate is larger when the components are less similar with respect to their boiling point temperature depending also on the activity coefficients of the chemical components in the condensate liquid. This reduction is accentuated at lower temperature differences between the cooled plate and the vapor at the bulk. A noncondensable gas causes a considerable decrease in the heat transfer even when it is present at the bulk of the vapor in very small percentage. However, its relative effect on the mixture is less than on a single component. Suction and forced convection may improve condensation rate although usually not as much as in the case of one component with a noncondensable gas.

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REFERENCES

Bird, R. B., Stewart, W. E. & Lightfoot, E. N. 1960 *Transport Phenomena*. Wiley.

Colburn, A. P. & Drew, T. B. 1937 The Condensation of mixed vapors. *Trans. AIChE J.* **33**, 157.

DENNY, V. E. & SOUTH III, Y. 1972 Effects of forced flow, noncondensables, and variable properties on film condensation of pure and binary vapors at the forward stagnation point of a horizontal cylinder, *Int. J. Heat Mass Transfer* **15**, 2133–2142.

DENNY, V. E. & JUSIONIS, V. J. 1972 Effects of forced flow and variable properties on binary film condensation, *Int. J. Heat Mass Transfer* **15**, 2143–2153.

HALA, E., PICK, J., FRIED, V. & VILIM, O. 1958 *Vapor–Liquid Equilibrium*, 1958. Translated by G. STANDART, Pergamon Press.

HALA, E., WICHTERLE, I., POLAK, J. & BOUBIK, T. 1968 *Vapour Liquid Equilibrium Data at Normal Pressures*. Pergamon Press.

HIRSCHFELDER, J. O., CURTIS, C. F. & BIRD, R. B. 1964 *Molecular Theory of Gases and Liquids*. Wiley.

MCADAMS, W. H. 1954 *Heat Transmission*, 3rd Ed. McGraw-Hill.

MINKOWYCZ, W. J. & SPARROW, E. M. 1966 Condensation heat transfer in the presence of noncondensables, interfacial resistance, superheating, variable properties and diffusion. *Int. J. Heat Mass Transfer* **9**, 1125–1144.

PERRY, J. H. 1963 *Chemical Engineerings' Handbook*, 4th Ed. McGraw-Hill.

REID, R. C. & SHERWOOD, T. K. 1966 *The Properties of Gases and Liquids*, 2nd Ed. McGraw-Hill.

ROSE, J. W. 1969 Condensation of vapour in the presence of a noncondensing gas. *Int. J. Heat Mass Transfer* **12**, 233–237.

SPARROW, E. M. & LIN, S. H. 1964 Condensation heat transfer in the presence of a noncondensable gas. *J. Heat Transfer* **c86**, 430–436.

SPARROW, E. M., MINKOWYCZ, W. J. & SADDY, M. 1967 Forced convection condensation in the presence of noncondensables and interfacial resistance. *Int. J. Heat Mass Transfer* **10**, 1829–1845.

SPARROW, E. M. & MARSCHALL, E. 1969 Binary gravity flow film condensation. *J. Heat Transfer* **91**, 205–211.

TAITEL, Y. & TAMIR, A. 1969 Condensation in the presence of a non-condensable gas in direct contact. *Int. J. Heat Mass Transfer* **12**, 1157–1169.

TAITEL, Y. & TAMIR, A. 1974 Multicomponent boundary layer characteristics—use of the Reference State. *Int. J. Heat Mass Transfer* (to be published).

TAMIR, A. & TAITEL, Y. 1971 Improving condensation rates by interfacial suction and forced convection in the presence of noncondensable gases. *Israel J. Technology* **9**, 69–81.

VDI Warmearlas 1953. Deutcher Ingenieur.

Résumé—Le modèle classique de Nusselt est èteu du au processus de condensation d'un mélange de vapeurs à plusieurs constituants. Les vapeurs condensées peuvent soit être miscibles à l'état liquide soit contenir des gaz non condensables.

La réduction du taux de condensation due à l'accumulation d'un gaz non condensable ou de constituants plus volatils près de l'interface de condensation est mise en évidence avec des systèmes à trois constituants, air–eau–méthanol et eau–méthanol–acétone. Les effets de l'aspiration à travers l'interface de condensation et de la convection forcée sont également pris en considération.

La solution analytique utilise la loi de Fick "généralisée" pour les systèmes à plusieurs constituants. La résolution exacte et la résolution approchée par la méthode intégrale sont toutes deux mises en oeuvre. La précision de la méthode intégrale se révèle remarquablement bonne.

Auzug—Das klassische Nusseltsche Modell wird verallgemeinert, um den Kondensationsprozess eines Vielstoff-Dampfgemisches einzuschliessen. Dabei koennen die kondensierenden Daempfe entweder im fluessigen Zustand mischbar sein, oder nicht kondensierbare Gase enthalten. Die Verringerung der Kondensationsgeschwindigkeit durch die Ansammlung eines nicht kondensierbaren Gases, oder von den fluechtigeren Komponenten, in der Naehe der Kondensatgrenzflaeche wird fuer Dreikomponentensysteme—Methanol–Wasser–Luft und Azeton–Methanol–Wasser—aufgezeigt. Saugwirkungen in der Grenzflaeche und Effekte erzwungener Konvektion sind ebenfalls eingeschlossen. Die analytische Loesung schliesst das "verallgemeinerte" Ficksche Gesetz fuer ein Vielkomponentensystem ein, und es werden sowohl exakte wie auch Naeherungsloesungen, die auf Integralmethoden beruhen, angewandt. Die Genauigkeit der Integralmethode erweist sich als bemerkenswert hoch.

Резюме—классическая теория Нуссельта обобщена для процессов конденсации в многокомпонентных смесях паров. Конденсированные пары могут быть как способными к смешению в их жидких состояниях, так и содерЖать неспособные к конденсации газы.

Понижение скорости конденсации происходит по причине накопления вблизи поверхности конденсата неспособного к смешению газа или более летучего компонента, что показано для трехкомпонентных систем: метилового спирта–воды–воздуха и ацетона–метилового спирта–воды. Учтено также влияние поверхностного поглощения и принудительной конвекции.

Аналитическое решение включает "обобшенный" закон фика для многокомпонентной системы, приложены как точное, так и приближенное интегральное решения. Точность интегрального метода представляется значительно лучшей.